Sec 1.5 Geometric Properties of Linear Functions

Interpreting the Parameters of a Linear Function

Ex. With time, t, in years, the populations of four towns, P_A, P_B, P_C, and P_D, are given by the following formulas:

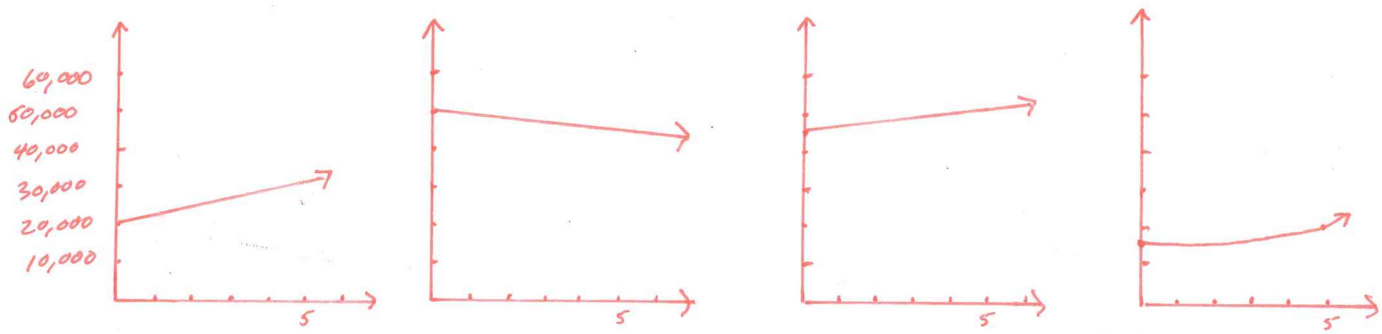
$$P_A = 20{,}000 + 1600t,\ P_B = 50{,}000 - 300t,\ P_C = 650t + 45{,}000,\ P_D = 15{,}000(1.07)^t.$$

(a) Which populations are represented by linear functions? A, B, C

(b) Describe in words what each linear model tells you about that town's population. Which town starts out with the most people? Which town is growing fastest?

C. Graph the functions above and explain how you can determine from the graph that they are linear functions (or are not). LINEAR → STRAIGHT LINE

A - The population starts at 20,000 and increases by 1600 each year.
B - The population starts at 50,000 and decreases by 300 each year.
C - The population starts at 45,000 and increases by 650 per year.



The Effect of the Parameters on the Graph of a Linear Function

Let $y = b + m\,x$. Then the graph of y against x is a line.

- The y-intercept, b, tells us where the line crosses the y-axis.
- If the slope, m, is positive, the line climbs from left to right. If the slope, m, is negative, the line falls from left to right.
- The slope, m, tells us how fast the line is climbing or falling.
- The larger the magnitude of m (either positive or negative), the steeper the graph of f.

Intersections of Two Lines

Ex. The cost in dollars of renting a car for a day from three different rental agencies and driving it d miles is given by the following functions:

$$C_1 = 50 + 0.10d \qquad C_2 = 30 + 0.20d \qquad C_3 = 0.50d.$$

(a) Describe in words the daily rental arrangements made by each of these three agencies.

(b) Which agency is cheapest?

$.50d = 30 + .20d$
$.30d = 30$
$d = 100$ miles

C_1 - Fifty dollar flat fee plus \$.10 per mile.
C_2 - Thirty dollar flat fee plus .20 per mile.
C_3 - No flat fee, fifty cents per mile.

$50 + .1d = 30 + .2d$
$20 = .1d$
$200m = = d$

C_3 when $d < 100$ $\quad C_2$ when $100 < d < 200$ $\quad C_3$ when $d > 200$

Equations of Horizontal and Vertical Lines

For any constant k:

- The graph of the equation y = k is a horizontal line and its slope is zero.
- The graph of the equation x = k is a vertical line and its slope is undefined.

Ex. Explain why the equation y = 4 represents a horizontal line~~s~~ and the equation x = 4 represents a vertical line.

y=4 means y=4 at every single point, with a varying x-value, which creates a horizontal line. x=4 means that the x-value is always 4 with varying y-values, which creates a vertical line.

Ex. Find the equations of d-g.

d. slope of 3 and contains the point (2, 4)

$4 = 3 \cdot 2 + b$
$4 = 6 + b$
$-2 = b$
$y = 3x - 2$

e. the line containing (2,3) and (-4, 5)

$m = \frac{5-3}{-4-2} = \frac{2}{-6} = -\frac{1}{3}$

$3 = -\frac{1}{3}(2) + b$
$3 = -\frac{2}{3} + b$
$3\frac{2}{3} = b$
$y = -\frac{1}{3}x + 3\frac{2}{3}$

f. slope of 3/4 and y-int of –2

$y = \frac{3}{4}x - 2$

g. (3, 2) and (4, 2)

$\frac{2-2}{4-3} = \frac{0}{1}$ $\quad y = 2$

Ex. Find the slope and intercept and then graph 8x – 2y = 6.

$-2y = -8x + 6$
$y = 4x - 3$

Parallel Lines – will always have the same or equal slope, y-intercepts will be different

Perpendicular lines – will always have negative reciprocals for slopes

Ex. Show that the following are either parallel or perpendicular lines or neither.

a. $3x + 2y = 4$ and $4x - 6y = 10$

$2y = 4 - 3x$ $\quad$ $-6y = -4x + 10$

$y = 2 - \frac{3}{2}x$ $\quad$ $y = \frac{2}{3}x - \frac{5}{3}$

Perpendicular

b. $2x + 3y = 6$ and $4x + 6y = 0$

$3y = -2x + 6$ $\quad$ $6y = -4x$

$y = -\frac{2}{3}x + 2$ $\quad$ $y = -\frac{2}{3}x$

Parallel

Ex. Find the equation of a line that contains (3, 1) and is parallel to $2x + y = 6$.

$y = -2x + 6$

$1 = -2(3) + b$

$1 = -6 + b$

$7 = b$

$y = -2x + 7$

Ex. Find the equation of a line that contains (3, 1) and is perpendicular to $2x + y = 6$.

$y = -2x + 6$

$m = \frac{1}{2}$

$1 = \frac{1}{2}(3) + b$

$1 = \frac{3}{2} + b$

$-\frac{1}{2} = b$

$y = \frac{1}{2}x - \frac{1}{2}$

HW: pg 42-44 #2, 3, 5, 7, 8, 13, 15, 16, 18, ~~19~~, 20, 23, ~~25~~, ~~28~~, 29, ~~30~~, 31 equation only